Analysis of the Effects of Surface Pitting and Wear on the Vibrations of a Gear Transmission System

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Abstract

A comprehensive procedure to simulate and analyze the vibrations in a gear transmission system with surface pitting, wear, and partial tooth fracture of the gear teeth is presented. An analytical model was developed where the effects of surface pitting and wear of the gear tooth were simulated by phase and magnitude changes in the gear mesh stiffness. Changes in the gear mesh stiffness were incorporated into each gear-shaft model during the global dynamic simulation of the system. The overall dynamics of the system were evaluated by solving for the transient dynamics of each shaft system simultaneously with the vibration of the gearbox structure. In order to reduce the number of degrees-of-freedom in the system, a modal synthesis procedure was used in the global transient dynamic analysis of the overall transmission system. An FFT procedure was used to transform the averaged time signal into the frequency domain for signature analysis. In addition, the Wigner-Ville distribution was also introduced to examine the gear vibration in the joint time-frequency domain for vibration pattern recognition. Experimental results obtained from a gear fatigue test rig at NASA Lewis Research Center were used to evalu-

1. Introduction

In the last two decades, problems arising from excessive wear and gear tooth surface pitting in gear transmission systems have been of increasing concern for a variety of gear users. At increased power and higher speeds, gear wear and fatigue failures are major concerns in aerospace applications. Although regular visual inspections and preventive maintenance can help to reduce the failure rate of gear systems, the cost and down time required make such programs inefficient and uneconomical.

Vibration signature analysis methodologies are being developed to non-intrusively examine the health and wear of gear transmission systems. Using spectral analysis, the amplitude of the frequency spectrum of the measured vibration signal is calculated and displayed in a continuous manner. However, the spectral analysis technique is difficult to apply in a highly complex system where the large number of spectral lines often makes it difficult to detect significant changes in the spectrum. Another methodology is the joint time-frequency approach which applies the Wigner-Ville distribution (WVD) [1-3] on the time vibration signal of the system. Unlike the regular Fourier transform process, the WVD provides an instantaneous frequency spectrum of the system at any instant throughout the sampling period (while FFT provides a averaged frequency spectrum of the total sampling period). The spectral density of the fundamental exciting frequency and its sidebands change as the shaft rotates through a complete revolution. Some success has been achieved in applying the WVD concept in the health monitoring of gear transmission systems [3-6]. However, a complete vibration signature database is needed for development of an effective pattern recognition scheme. In order to populate such databases, the development of an accurate analytical procedure to predict vibrations in gear systems due to wear and fatigue failure is necessary.

The objective of this paper is to develop a comprehensive procedure to simulate and analyze the vibration in a gear transmission system with effects of surface pitting and wear of the gear teeth under normal operating conditions. The effects of changes in magnitude and phase of the mesh stiffness at one particular tooth or a number of consecutive teeth were evaluated in order to simulate the effects of surface pitting and wear. The effects of these localized changes in the gear mesh were incorporated into each gear-rotor model for the dynamic simulation [7-9]. The dynamics of each gear-rotor system were coupled with each other through the gear mesh interacting forces. The coupling between the rotors and the casing structure were generated through the bearing support forces. The global vibrations of the system were evaluated by solving the transient dynamics of each rotor system simultaneously with the vibration of the casing. In order to minimize the computational effort, the number of degrees-of-freedom of the system were reduced by using a modal synthesis procedure [7,8]. Experimental vibration results obtained from a gear fatigue test rig at NASA Lewis Research Center [11] were used to verify the analytical procedure.

2. Solution Procedures

2.1 Dynamics of the Gear-Shaft Configuration and the Gearbox System

The dynamics of the ith individual gear-shaft system can be evaluated through the equations of motion for the vibrations of a individual rotor-bearing-gear system as shown in Figure 1[7,8], given in matrix form, as

$$[M]\{\ddot{W}_i\} + [K_s]\{W_i\} = \{F_{bi}(t)\} + \{F_{gi}(t)\} + \{F_{ui}(t)\} \quad (1)$$

where [M] and [K_s] are respectively the mass and shaft stiffness matrices of the rotor, $\{W_i\}$ is the general displacement

vector of the ith rotor in the its local coordinate system, and, $\{F_{bi}(t)\}$, $\{F_{gi}(t)\}$, and $\{F_{ui}(t)\}$ are respectively the force vectors acting on the ith rotor system due to bearing forces, gear mesh interactions, and mass-imbalances.

The equations of motion of the gearbox with p rotor systems can be expressed as

$$\left[M_c\right]\left\{\ddot{W}_c\right\}+\left[K_c\right]\left\{W_c\right\}=\sum_{i=1}^{p}\left[T_{ci}\right]\left\{F_{bi}(t)\right\} \quad (2)$$

where $[T_{ci}]$ represents the coordinate transformation between the ith rotor and the gearbox.

2.2 Evaluation of Bearing Forces

The bearing forces $\{F_{bi}(t)\}$ for the ith rotor can be evaluated as

$$\left\{F_{bi}(t)\right\}=\left[C_{bi}\right]\left(\left\{\dot{W}_i\right\}-\left[T_{ic}\right]\left\{\dot{W}_{ci}\right\}\right)$$
$$+\left[K_{bi}\right]\left(\left\{W_i\right\}-\left[T_{ic}\right]\left\{\dot{W}_{ci}\right\}\right) \quad (3)$$

where $[C_{bi}]$ and $[K_{bi}]$ are respectively the damping and stiffness of the bearing, $[T_{ic}]$ is the coordinate transformation matrix for the gearbox with respect to the ith rotor, and W_{ci} are the casing displacements at the rotor locations.

2.3 Evaluation of Gear Forces

The gear forces generated from the gear mesh interaction[12] can be written as

$$\left\{F_{gi}(t)\right\}=\left\{F_{ri}(t)\right\}+\left\{F_{ti}(t)\right\} \quad (4)$$

where $\{F_{ri}(t)\}$ is the vector containing the gear forces and moments resulting from the relative rotation between the two mating gears and $\{F_{ti}(t)\}$ is the vector containing gear forces and moments due to the translational motion between the two gears.

2.4 Modal Synthesis Procedure

In order to calculate the transient and steady state dynamics of the system, the coupled rotor and casing equations of motion must be solved simultaneously. To minimize the computational effort, the modal transformation[7,8] procedure will be applied to reduce the degrees of freedom of the global equations of motion. Using m undamped mode shapes of the ith rotor system $[\phi_{i1}, \phi_{i2}, \phi_{i3}, ..., \phi_{im}]$ and m_c undamped mode shapes of the gearbox $[\phi_{c1}, \phi_{c1}, \phi_{c3}, ..., \phi_{cmc}]$, the rotor displacement for the ith rotor can be written as

$$\left\{W_i\right\}=\sum_{j=1}^{m}A_{ij}\left\{\phi_{ij}\right\} \quad (5)$$

and, similarly, the gearbox displacements as

$$\left\{W_c\right\}=\left[\phi_c\right]\left\{A_c\right\} \quad (6)$$

where $\{A_i\}$ and $\{A_c\}$ are the modal time functions of the ith rotor and the gearbox respectively. Using the expansion in equation (5), the equations of motion for the ith rotor in equation (1) can be written as

$$[M]\left[\phi_c\right]\left\{\ddot{A}_i\right\}+\left[K_s\right]\left[\phi_i\right]\left\{A_i\right\}$$
$$=\left\{F_{bi}(t)\right\}+\left\{F_{gi}(t)\right\}+\left\{F_{ui}(t)\right\} \quad (7)$$

Premultiplying by $[\phi_i]^T$ and using the orthogonality conditions of the mode shapes [7], the ith rotor equations of motion can be written as

$$\left\{\ddot{A}_i\right\}+\left[\Lambda^2\right]\left\{A_i\right\}=\left\{\overline{F}_{bi}\right\}+\left\{\overline{F}_{gi}\right\}+\left\{\overline{F}_{ui}\right\} \quad (8)$$

where $[\Lambda^2]$ is the diagonal matrix of the squares of the natural frequencies of the system.

For the gearbox system, a similar transformation is carried out as equation (2) can be written as

$$\left\{\ddot{A}_c\right\}+\left[\Lambda_c^2\right]\left\{A_c\right\}=\left\{\overline{F}_{cb}\right\} \quad (9)$$

For a system of k rotors, equation (8) can be repeated k times and solved with the casing equation (9) simultaneously for the modal accelerations $\{A_i\}$ and $\{A_c\}$. A numerical integration scheme is used to integrate the accelerations to obtain velocities and displacements at each time step for transient calculations [7].

3. Signature Analysis of Vibration Signal

3.1 Frequency Domain Analysis

The frequency spectrum analysis is used by applying a discrete Fourier Transform on the average time signal x(t) such that the spectral components are

$$X(k)=T\sum_{i=0}^{N-1}x(t)\exp\left(\frac{-j2\pi ik}{N}\right) \quad (10)$$

where x(t) is the time averaged of the vibration signal W(t) and T is the sampling interval. The frequency components are examined in the frequency domain and compared with those obtained at various stages of the fault development in the experimental gear test rig.

3.2 Joint Time-Frequency Analysis : The Wigner-Ville Distribution

To examine the vibration signal in a joint time-frequency domain, the Wigner-Ville method [1-3] is used in this study.

The Wigner-Ville distribution will provide an inter-domain relationship between time and frequency during the period of the time data window. The WVD (Wigner-Ville Distribution) can be written as:

$$WV(t,f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right) x*\left(t - \frac{\tau}{2}\right) e^{-j2\pi ft} \, dt \quad (11)$$

where WV(t, f) is the Wigner-Ville distribution in both the time domain t and the frequency domain f. To allow sampling at the Nyquist rate and eliminate the concentration of energy around the frequency origin due to the cross product between negative and positive frequency [1,2], the analytic signal was used in evaluating the WVD. The analytic signal s(t) is defined as

$$S(t) = x(t) + jH\big[x(t)\big] \quad (12)$$

Where H[x(t)] is the Hilbert transform of x(t).

In order to avoid a repetition in the time domain WVD, a weighting function[4] is added to the time data before the evaluation process. Such process may decrease the resolution of the distribution, but it will eliminate the repetition of peaks in the time domain, and, thus the interpretation of the result will be substantially easier.

4. Description of Experimental Study

The experiment was performed on the spiral bevel gear fatigue test rig [11], as illustrated in Figure 2, at the NASA Lewis Research Center. The primary purpose of this rig is to study the effects of gear tooth design, gear materials, and lubricants on the fatigue strength of aircraft quality gears. Because spiral bevel gears are used extensively in helicopter transmissions to transfer power between non-parallel intersecting shafts, the use of this fatigue rig for diagnostic studies is practical. Vibration data from an accelerometer mounted on the pinion shaft bearing housing was captured by an analog to digital conversion board. The 12-tooth test pinion, and the 36-tooth gear have a 35 degree spiral angle, a 1 in. face with, a 90 degree shaft angle, and 22.5 degree pressure angle. The pinion transmits 720 hp at a nominal speed of 14,400 rpm. The test rig was stopped several times during the test for gear damage inspection. The test was concluded at 17.8 operational hours when a broken tooth was detected visually during one of the shutdowns.

Pictures of tooth damage on the pinion at various stages in the test are shown in Figure 3. At the first rig shut-down, at about 5.5 hours into the test, a small pit was observed on one of the teeth on the test pinion, as illustrated in Figure 3A. The test was stopped again at approximately 12 hours and the pitted area spread to cover approximately 75% of the face of the pinion tooth, as seen in Figure 3B. In addition, pitting started to appear on the adjacent teeth. Figure 3B shows the pinion at the end of the test, at 17.8 hours. It was found that one of the three heavily pitted pinion teeth had experienced a tooth breakage, losing one third of the tooth, as shown in the figure.

5. Discussions of Results

To study the effects of gear tooth pitting and wear on the dynamics of the rotor system, the numerical simulation procedure described above was used to model the vibrations of the pinion gear in the test rig. During the experimental study, vertical direction vibration signals from the pinion gear are time synchronously averaged for spectral analysis and analysis using the joint time-frequency distribution(WVD). In order to perform an accurate comparison, the averaged time signal from the vertical vibration of the pinion gear is also generated using the numerical model. During these simulations, approximate gear mesh stiffness models are developed to simulated the effects of wear and pitting of the pinion tooth on the dynamics of the system.

As it has been established, the changes due to gear tooth wear or failure can be represented by the amplitude and phase changes in vibration, which, in turn, can be represented by magnitude and phase changes in mesh stiffness[5,6]. To demonstrate the effects of mesh stiffness change on gear vibration, the variation of the mesh stiffness model used for this study is given in Fig 4. The "undamaged" configuration of the mesh stiffness is given by 0 degree phase change(Fig. 4A), and 0% amplitude reduction(Fig. 4B).During the wear and pitting process, two types of stiffness changes are examined, i.e., the phase changes, shown in Figure 4A, and the amplitude changes, shown in Figure 4B. Figures 5 and 6 show the time, frequency, and joint time-frequency analysis(WVD) of the pinion gear vibration signals with the approximated changes in gear mesh stiffness.

Figure 5 shows the effects of mesh stiffness phase changes in the WVD representation of the predicted vibration signal. As seen in Figure 5, a phase change in the mesh stiffness at the 6th tooth of the 12-teeth pinion resulted in a temporary increase of amplitude and phase of the pinion vibration time signal during the 6th tooth pass location. As the phase shift in the mesh stiffness increases, from 1.5 degrees to 4.5 degrees, the changes in amplitude and phase in the vibration signal become more pronounced. In the frequency spectra, this change in mesh stiffness will result in the increase of the amplitude in the sideband frequencies. However, as discussed earlier, although the frequency spectrum provides good indications of the existence of the non-synchronous components, it can not distinguish the time locations of their occurrences. The joint time-frequency analysis using WVD, shows the existence of various frequency components as the pinion rotates through a complete revolution of 360 degrees. Note that, in this case, the WVD shows a continuous excitation of the mesh frequency (12 x rotational speed) throughout the complete 360-degree revolution while subsynchronous components of 8 times, 4 times, and 1 times rotational speed are occurring at the 6th and 7th tooth pass locations.

Figure 6 shows the effects of reductions in mesh stiffness at the 6th gear location. With the reduction of mesh stiffness, a substantial change in the vibration at the 6th tooth pass location (150 - 180 degrees) is observed. Note that in the case of 50% stiffness reduction, the time vibration amplitude

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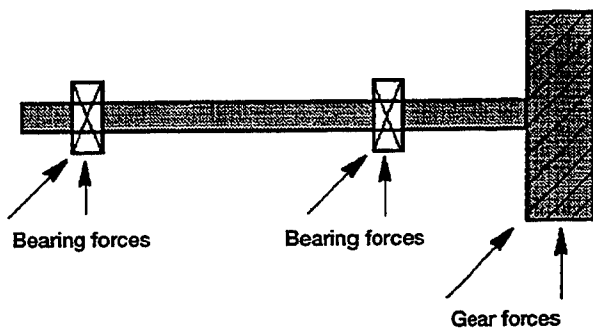
Figure 1.—Schematic of the rotor-gear bearing system.

Bearing forces

Bearing forces

Gear forces
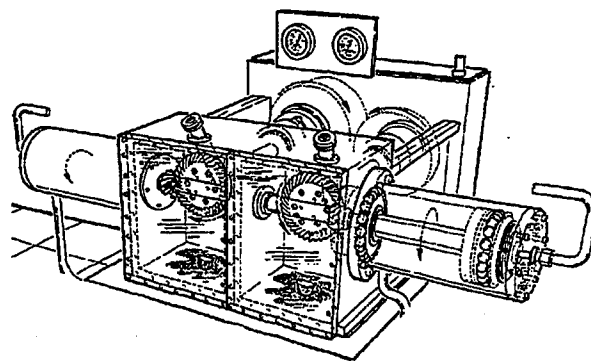
Figure 2.—Picture of the bevel gear test rig in NASA Lewis Research Center.
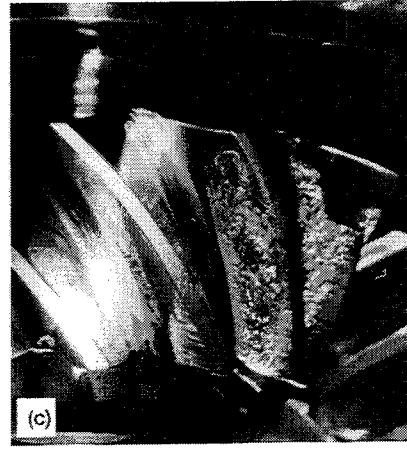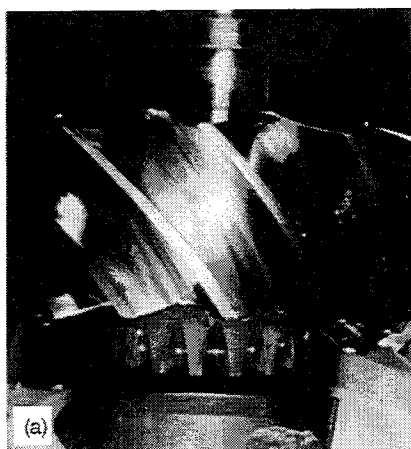

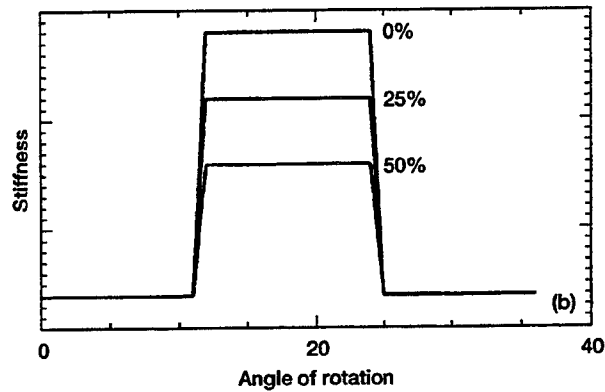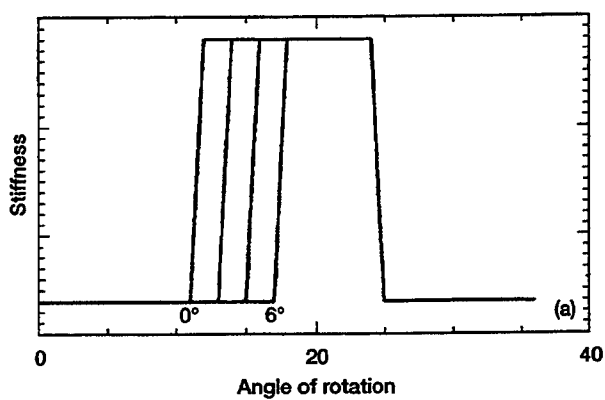Figure 3.—Pictures of the damaged pinion teeth. (a) 5.5 hr. (b) 12 hr. (c) 17.8 hr.

Figure 4.—Stiffness changes in the gear mesh model.

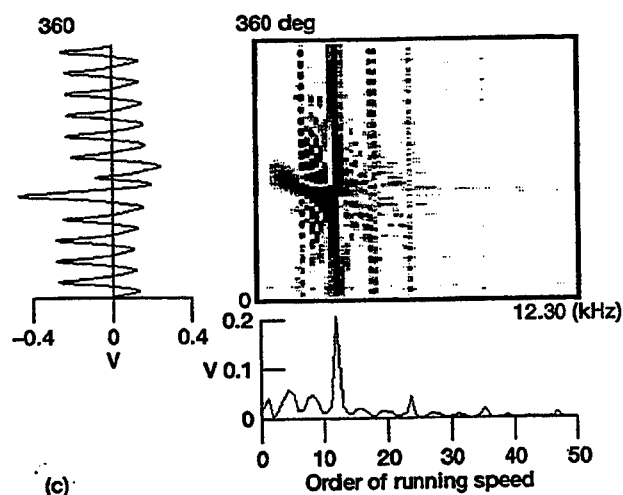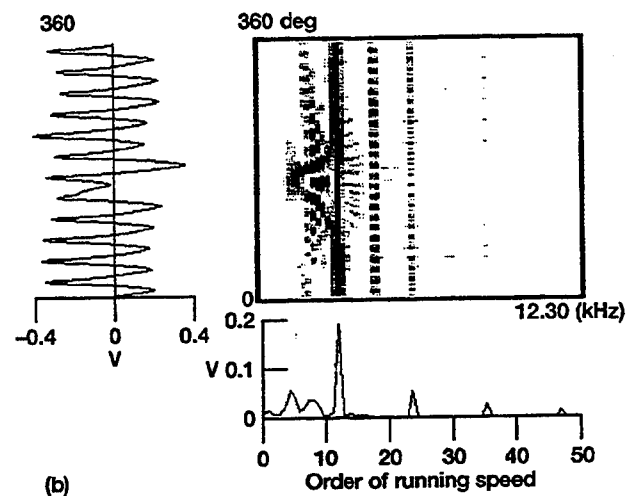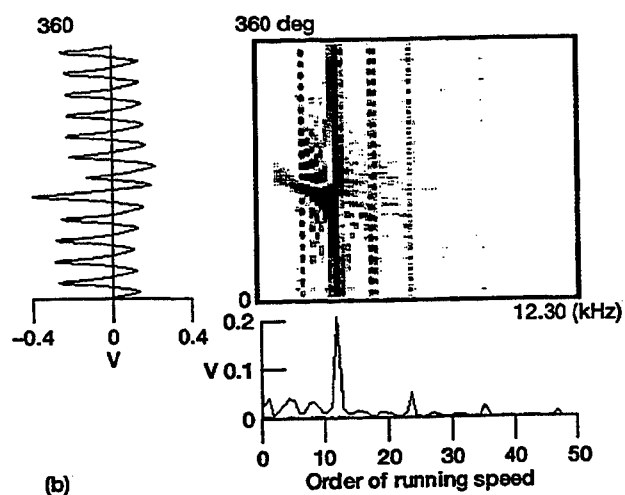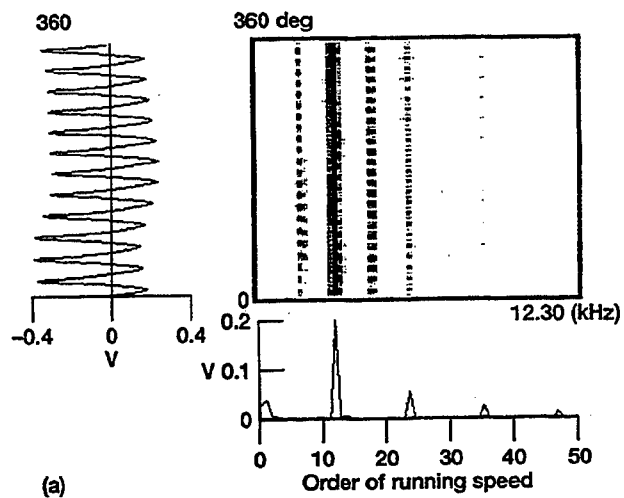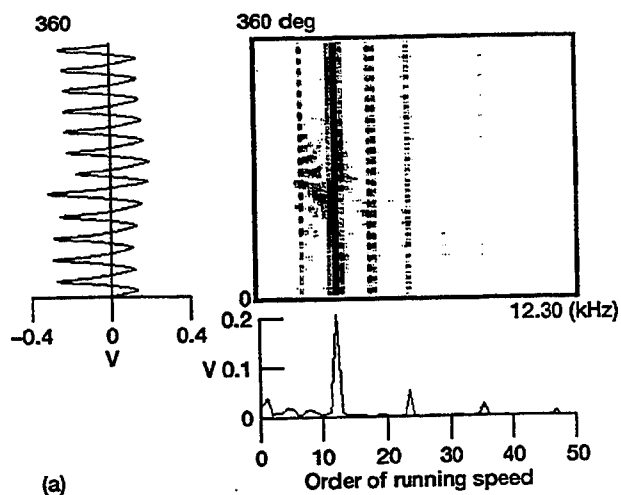Figure 5.—Simulated pinion vibration signature due to phase change in the gear mesh stiffness. (a) 1.5°. (b) 3.0°. (c) 4.5°.
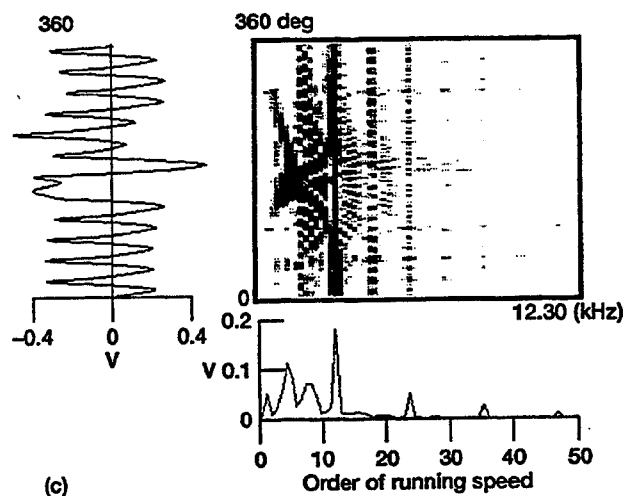
Figure 6.—Simulated pinion vibration signature due to amplitude change in the gear mesh stiffness. (a) 0% reduction. (b) 25% reduction. (c) 50% reduction.
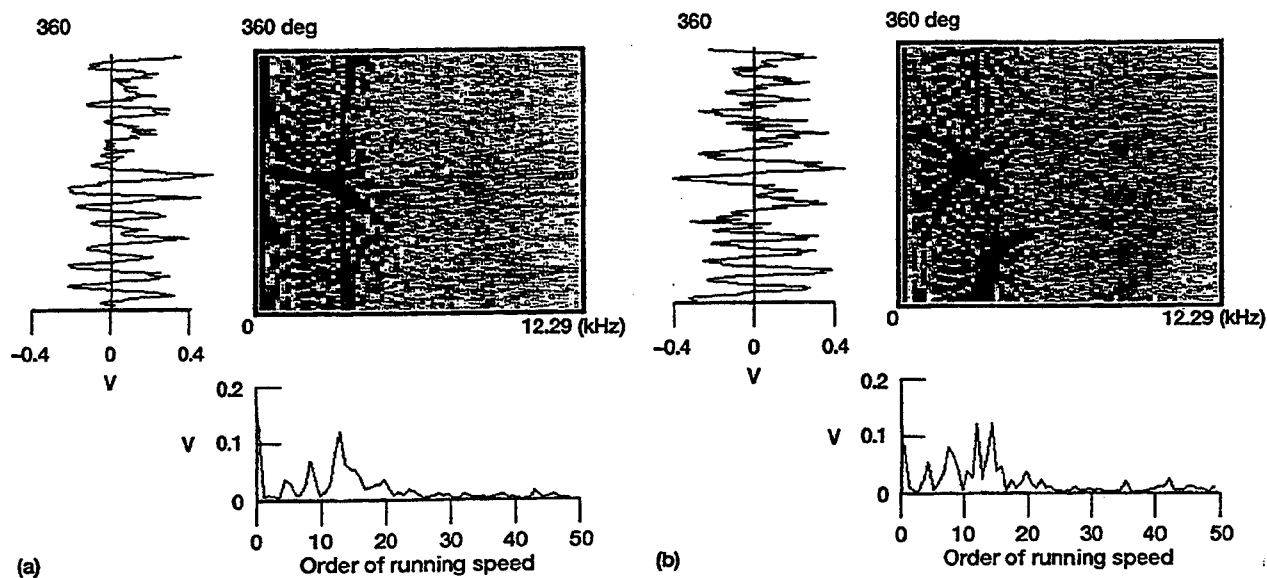
Figure 7.—Experimental pinion vibration signature due to damage on pinion teeth due to wear and pitting. (a) Single tooth, (12 hr). (b) Three teeth, (17.8 hr).
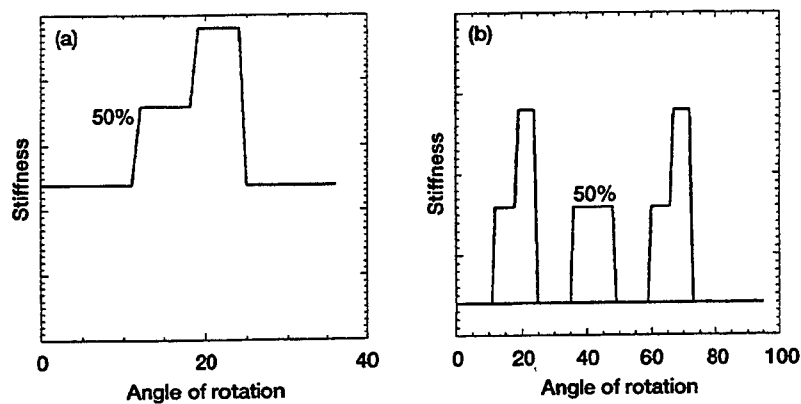

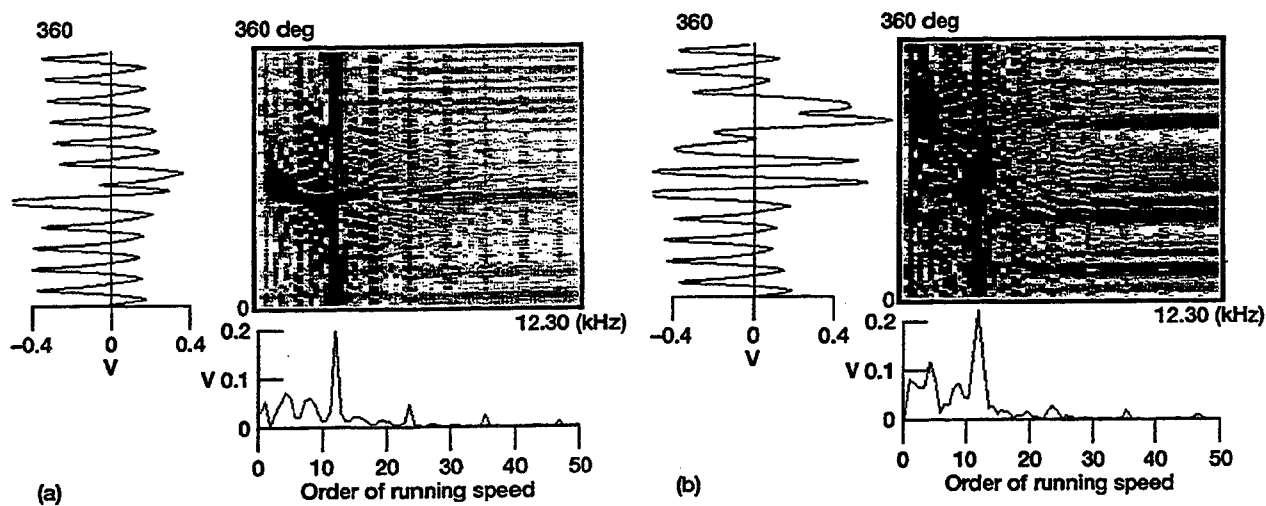Figure 8.—Gear mesh stiffness model to simulate damages in pinion gear teeth.

Figure 9.—Numerically simulated pinion vibration signature due to damage on pinion teeth due to wear and pitting. (a) Single tooth. (b) Three teeth.

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13. ABSTRACT (Maximum 200 words)

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